Erratum: Quantum phase-space representation for curved configuration spaces [Phys. Rev. A 88, 062117 (2013)]

Clemens Gneiting, Timo Fischer, and Klaus Hornberger

(Received 2 December 2022; published 23 December 2022)

DOI: 10.1103/PhysRevA.106.069904

Section VI of the original article treats the Wigner-Weyl representation of the particle on the sphere. In Sec. VI C the equation of motion for the Wigner function (85) of a quantum state ρ is presented to demonstrate that the semiclassical limit reduces it to the corresponding classical Liouville equation. Some terms are missing in Eq. (85), which arise due to the boundedness of the configuration space. These expressions vanish semiclassically so that all equations except (85) and all conclusions remain valid.

Specifically, on the right-hand side of Eq. (85) the following terms must be added:

$$-\frac{2\hbar(-1)^{m_\vartheta}}{MR^2\pi}\partial_\vartheta A(\vartheta,\varphi;m_\varphi)-\frac{\hbar(-1)^{m_\varphi}}{MR^2\pi}\sum_{n=0}^{\infty}\frac{\partial^{2n}g^{\varphi\varphi}(\vartheta,\varphi)}{\partial\vartheta^{2n}}(-1)^n\frac{(\hbar/2)^{2n}}{(2n)!}\frac{1}{(2\hbar)^{2n}}\sum_{m'_\vartheta\in\mathbb{Z}}\delta^{(2n)}_{m_\vartheta-m'_\vartheta}\partial_\varphi B(\vartheta,\varphi;m'_\vartheta)$$
$$+\frac{\hbar(-1)^{m_\varphi}}{2MR^2\pi}\sum_{n=0}^{\infty}\frac{\partial^{2n+1}g^{\varphi\varphi}(\vartheta,\varphi)}{\partial\vartheta^{2n+1}}(-1)^n\frac{(\hbar/2)^{2n}}{(2n+1)!}\frac{1}{(2\hbar)^{2n+1}}\sum_{m'_\vartheta\in\mathbb{Z}}\delta^{(2n+1)}_{m_\vartheta-m'_\vartheta}[\hbar m_\varphi B(\vartheta,\varphi;m_\vartheta)+C(\vartheta,\varphi;m'_\vartheta)].\quad(1)$$

They involve the limits

$$A(\vartheta,\varphi;m_\varphi)=\lim_{\vartheta'\uparrow\frac{\pi}{2}}\sum_{m'_\vartheta\in\mathbb{Z}}\sin(2m'_\vartheta\vartheta')W(\vartheta,\varphi,m'_\vartheta,m_\varphi),\quad(2a)$$

$$B(\vartheta,\varphi;m'_\vartheta)=\lim_{\varphi'\uparrow\pi}\sum_{m'_\varphi\in\mathbb{Z}}\sin(m'_\varphi\varphi')W(\vartheta,\varphi,m'_\vartheta,m'_\varphi),\quad(2b)$$

$$C(\vartheta,\varphi;m'_\vartheta)=\lim_{\varphi'\uparrow\pi}\sum_{m'_\varphi\in\mathbb{Z}}\hbar m'_\varphi\sin(m'_\varphi\varphi')W(\vartheta,\varphi,m'_\vartheta,m'_\varphi).\quad(2c)$$

These expressions do not vanish, in general. This is because the convergence of the series may not be absolute so that one is not necessarily allowed to interchange the limits and the summations. Indeed, for quantum states displaying spatial coherences between antipodal positions they may yield a finite contribution.

We emphasize that this correction does not affect the rest of the article. Equation (85) still holds true for sufficiently classical states, whose angular coherences $c_{\varphi,\vartheta}(\varphi',\vartheta')=\langle\varphi_-,\vartheta_-|\rho|\varphi_+,\vartheta_+\rangle$ decay sufficiently fast so that they do not "feel" the fact that they are living on a finite configuration space: The limits (2) vanish if $c_{\varphi,\vartheta}=0$ for $|\varphi'|>\pi-\epsilon$ and $|\vartheta'|>\pi/2-\epsilon$ with $\epsilon>0$. [Recall that $\varphi_\pm=(\varphi\pm\varphi'/2)\bmod 2\pi$, $\vartheta_\pm=(\vartheta\pm\vartheta'/2)\bmod\pi$.]

As a further correction, in the arguments of the sinc functions of Eq. (82) the terms $m'_\vartheta-m''_\vartheta$ and $m'_\varphi-m''_\varphi$ must be replaced by $m'_\vartheta+m''_\vartheta$ and $m'_\varphi+m''_\varphi$, respectively.

