Supplementary Information: Quantum electromechanics with levitated nanoparticles

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In this Supplementary Information we provide a discussion of the impact of particle dipole moments on the measurement outcome (I.) and we discuss the most important sources of decoherence in the proposed setup (II.).

SUPPLEMENTARY NOTE I: IMPACT OF DIPOLE MOMENTS

For the proposed parameters, including a small but finite dipole moment, the coupling between nanoparticle rotations and Cooper-pair box is so small that the orientation state after the pulse sequence almost perfectly overlaps with the state before the pulse sequence. The ensuing reduction of the interference signal is therefore negligible.

A quantitatively more rigorous version of this argument, consists of five steps:

a) First, the initial correlations between centre-of-mass state and rotation state are negligible.
b) Second, these degrees of freedom do not interact directly and thus only become correlated via the qubit.
c) Third, the rotation is very slow compared to the timescale of the experiment and thus its kinetic phase plays no role.
d) Fourth, the resulting decay of the rotational interference signal is dominated by the rotational coupling to the qubit, which has a negligible effect on the timescale of the experiment.
e) The relative phase on the rotation state induced by the external potential is negligible.

a) Initial correlations:

The spatial width of the initially thermal state is quantified by $z_{\rm th} = \sqrt{2k_{\rm B}T/M\omega^2} \approx 2.9 \cdot 10^{-8}$ m, and thus the harmonic contribution in Eq. (1) clearly dominates the coupling to the rotations since $p/qz_{\rm th} \approx 0.03 \ll 1$, $p^2M/I{\rm q}^2 \approx 0.007 \ll 1$ and ${\rm Qpz}_{\rm th}M/I\, q^2z_{\rm th}^2 \approx 0.1 \ll 1$. (This last parameter overestimates the coupling since it neglects that the dipole moment is strongly aligned with the trap axis.) Here we assumed a dipole moment of $p = |p| = 2000\,$eÅ, a value ten-fold higher than what we deem realistic.

Conversely, the rotational motion is not influenced by the centre of mass since $z_{\rm th}p/Q \approx 0.06 \ll 1$ and $z_{\rm th}pq\, I/MQ^2 \approx 0.02 \ll 1$, where ${\rm Q} = 1.5 \cdot 10^{-32}\,{\rm Cm}^2$ quantifies the quadrupole tensor $Q = Q(\boldsymbol{m} \otimes \boldsymbol{m} - \mathbb{1}/3)$ of the cylinder with homogeneous surface charge and symmetry axis $\boldsymbol{m}$.

The rotations and the centre-of-mass are thus initially uncorrelated, where the centre-of-mass is well approximated by a thermal state in the shifted harmonic potential while the rotations are given by a thermal state with potential

$$V_{\rm eff}(\beta) = \frac{3U_{\rm ac}^2p^2}{16Mz_0^4\Omega_{\rm ac}^2}\cos^2\beta + \frac{U_{\rm ac}^2}{16Iz_0^4\Omega_{\rm ac}^2} sin^2\beta\,(2pz_s + Q\,cos\,\beta)^2.$$

Here, we used that the dipole points into the direction of the particle symmetry axis and β denotes the angle between the particle and the trap symmetry axis.

b) Coupling to qubit:
Equation (18) describes that the Cooper-pair box adds two terms that are independent of the qubit state to the effective potential (1): First, the constant shift of magnitude $-2ekN(q\boldsymbol{r}+\boldsymbol{p})\cdot\boldsymbol{e_z}/C_\Sigma z_0$ by the second term in Eq. (18) shifts the potential to the new minimum $z=z_s$. Since this shift fulfils $p^2 z_{\mathrm{th}} z_{\mathrm{s}}/p^2 z_{\mathrm{s}}^2 \approx 0.025 \ll 1$ and $p^2 z_{\mathrm{s}} z_{\mathrm{th}} M/q^2 z_{\mathrm{th}}^2 I \approx 0.27,$ the initial rotational and translational state remains uncorrelated (again, the last parameter overestimates the coupling). Second, the third term of (18) slightly deforms the trapping potential, which is negligible in comparison to the effective potential (1). Thus, the dominant effect of the qubit on the rotation state is independent of the centre-of-mass coordinates and is described by the coupling term $-2ek\sigma^+\sigma^-\boldsymbol{p}\cdot\boldsymbol{e_z}/C_\Sigma z_0$.

During the pulse scheme, a centre-of-mass superposition also induces a rotational superposition via the coupling in the effective potential. However, for our parameters the rotational delocalization due to the CPB dominates by several orders of magnitude. Similarly, the spatial superposition induced by the rotational superposition is negligible. We can thus treat translation and rotation independently during the whole experiment so that the reduction of the interference visibility due to the rotations is given by the overlap between the two rotational interference branches.

c) Angular momentum kicks:
During the pulse sequence, the rotations evolve in the effective potential $V_{\mathrm{eff}}(\beta)$, so that the interaction picture dynamics of the pure rotor-qubit state $|\Phi\rangle$ are given by

$$i\hbar\,\partial_t|\Phi\rangle = e^{itH_0/\hbar}Ve^{itH_0/\hbar}|\Phi\rangle$$

Here, $H_0 = E_c\sigma^+\sigma^- + H_{rot}$ and $\mathrm{V} = \mathrm{V}_{\mathrm{eff}}(\beta) - 2\hbar\kappa_{\mathrm{rot}}\sigma^+\sigma^-\cos\beta$ with the rotational coupling frequency $\kappa_{rot} = ekp/\hbar z_0 C_\Sigma \approx 17.7$ MHz.

Treating the cylinder as a linear rotor and expanding $|\Phi\rangle$ in the combined qubit-angular momentum basis $|\sigma lm\rangle$ with the total and magnetic angular momentum quantum numbers l and m, yields a linearly coupled set of differential equations for the expansion coefficients $\Phi_{\sigma lm}$,

$$i\hbar\,\partial_t\Phi_{\sigma lm} = \sum_{\sigma' l' m'} exp\left(\frac{it}{\hbar}\left[\mathrm{E_c}(\delta_{\sigma 1}-\delta_{\sigma' 1}) + \frac{\hbar^2}{2\mathrm{I}}l(\mathrm{l}+1) - \frac{\hbar^2}{2\mathrm{I}}l'(l'+1)\right]\right)\langle\sigma lm|\mathrm{V}|\sigma'\mathrm{l'm'}\rangle\Phi_{\sigma' l' m'}.$$

The potential V only couples momentum states that maximally differ by $l-l' = \pm\Delta l = \pm 4$. Inserting the experimental timescale $t = t_1 = 21.7\,\mathrm{ns}$, the thermal angular momentum $l = 800$ and the moment of inertia $I = 2.48\cdot 10^{-37}\,\mathrm{kgm}^2$, shows that the oscillating phase in the above time evolution is negligibly small since

$$\frac{t}{\hbar}\left[\frac{\hbar^2}{2I}(l+4)(l+4+1) - \frac{\hbar^2}{2I}l(l+1)\right] = t\frac{\hbar}{2I}(8l+20) \approx 0.028 \ll 1,$$

for all realistically occupied l states.
The kinetic terms can thus be neglected, yielding the effective interaction picture Schrödinger equation

$$i\hbar\,\partial_t|\Phi\rangle = V|\Phi\rangle$$

describing that the particle does not rotate during the pulse sequence. Since $\hbar\kappa_{\rm rot}/k_B T \approx 0.13 < 1$ the rotations induced by the CPB do not violate this approximation. The dynamics describe a superposition of angular momentum kicks, $|\Phi(\mathrm{t})\rangle \approx \exp\left(-\frac{i}{\hbar}tV\right)|\Phi_0\rangle$, with a relative strength according to the orientation dependence of V.

d) Signal reduction:
For the thermal rotation state ρ_0 (and neglecting the centre of mass for the moment), the time evolution yields after the pulse sequence the qubit population

$$\langle\sigma^+\sigma^-\rangle = \frac{1}{4} + \frac{1}{4}e^{\frac{i}{\hbar}E_c t'}\mathrm{tr}\left[\rho_0 \mathrm{e}^{-2i\kappa_{\rm rot}t'\cos\beta}\right] + h.c.$$

With increasing time $t' = 2t_1 - 2t_2 + t_3$ the overlap $\mathrm{tr}\left[\rho_0 \mathrm{e}^{-2i\kappa_{\rm rot}t'\cos\beta}\right]$ and thus the interference visibility decreases.

The thermal rotation state includes bound and unbound rotational contributions. To get an upper limit for the visibility loss we calculate the measurement signal for the free momentum eigenstates $|\chi\rangle = |lm\rangle$. These states are completely localized in angular momentum space and thus the torque-induced reduction of their overlaps is maximal. In contrast, bound states are localized in the orientation space and thus broader in momentum space, so that their overlaps vanish slower.

For a free thermal rotor state, the qubit population can be calculated as

$$\langle\sigma^+\sigma^-\rangle = \frac{1}{2} + \frac{1}{2}\cos\left[\frac{E_c}{\hbar}(2t_1 - 2t_2 + t_3)\right]\mathrm{sinc}[2\kappa_{rot}(2t_1 - 2t_2 + t_3)].$$

This expression is independent of the rotor temperature, and the visibility loss is characterized by $s\mathrm{inc}[2\kappa_{rot}(2t_1 - 2t_2 + t_3)]$. For fixed $t_1 = 21.7$ ns and $t_2 = 65.1$ ns, and with t_3 between 80 ns and 87 ns (see Fig. 3) the visibility varies from 0.99 to 1, demonstrating that rotational dephasing is irrelevant for the measurement outcome.

e) Imprinted phase:
To unambiguously demonstrate that the rotations can be neglected for the proposed interference experiment we also have to show that the relative phase between the two centre-of-mass branches is always much larger than the corresponding rotational relative phase. This latter relative phase appears as an additional contribution in the above cosine function or in Eq. (25). The phase can be estimated semiclassically by approximating the extension of the CPB-induced orientational superposition, evaluating the resulting potential difference in an external homogeneous field E_{ext} and dividing by $\hbar$. This yields for the rotations

$$\Delta\varphi_{\rm rot} = \frac{keE_{ext}t^3}{3\mathrm{C}_\Sigma \mathrm{z}_0\hbar}\frac{p^2}{I}$$

and for the centre of mass

$$\Delta\varphi_{\rm cm} = \frac{keE_{ext}t^3}{3\mathrm{C}_\Sigma \mathrm{z}_0\hbar}\frac{q^2}{M}.$$

The latter result is consistent with Eq. (25) expanded for $\omega\tau \ll 1$ and thus validates this estimate. Since $p^2/I \approx 0.007 q^2/M$, the rotational phase is negligible and thus does not affect the measurement signal.

SUPPLEMENTARY NOTE II: DECOHERENCE MECHANISMS

The simulations shown in Fig. 3 take the most important decoherence mechanism, i.e. qubit dephasing, into account.

Specifically, the most important sources of decoherence and dephasing in our setup are in order of negative impact

a) **Qubit dephasing**: Decoherence of the qubit state is dominated by dephasing due to charge noise. This is described in the Methods and in the caption of Fig. 3, leading to a reduction of the visibility.

b) **Nanoparticle rotations**: Nanoparticle rotations and their correlations with the qubit and the centre-of-mass motion decrease the fringe visibility of the interference pattern. In our setup, this poses a condition on the maximally allowed dipole moment ($\lesssim 2000\,\mathrm{e\AA}$). A detailed discussion of this bound can be found above.

c) **Gas collisions**: Collisions with residual gas particles can quickly decohere the quantum state of the nanoparticle [8, NJP **12** 033015 (2010)]. The resulting decoherence rate is bounded from above by the frequency of collisions with gas particles. This total collision rate follows from integrating the thermal gas flux at temperature T and pressure p_g over the particle surface [PRE **97**, 052112 (2018)] as

$$\Gamma_{\text{coll}} = \frac{p_g R(R+L)}{\mathrm{m_g} k_B T}$$

For nitrogen ($m_g = 28$ amu) at room temperature and 10^{-4} mbar, one obtains $\Gamma_{\text{coll}} = 1.88 \cdot 10^5$/s for the proposed particle shape. Thus, on average only 0.016 gas collisions occur during the pulse sequence of 87 ns. Decoherence due to gas collisions can thus be safely neglected.

d) **Surface noise:** Fluctuating dipoles on the electrode surface can heat up the particle motion [61] and decohere the particle state. The heating rate $\Gamma_h = 170/s$ is given in the main text and calculated on the basis of [61]. The corresponding momentum diffusion coefficient $D = \hbar\omega\Gamma_h m$ can be used to estimate the spatial decoherence rate

$$\Gamma_{\text{dec}} = \frac{D}{\hbar^2}\Delta z^2.$$

We integrate the force due to one Cooper pair over the experimental time scale $\mathrm{t} = 87\,\mathrm{ns}$ to obtain an upper bound for the spatial superposition $\Delta z < t^2 ekq/mC_\Sigma z_0 \approx 8.5\,\mathrm{pm}$. The resulting decoherence is negligible during the experiment, $\Gamma_{\text{dec}}\mathrm{t} \approx 2.3 \cdot 10^{-9}$.

e) **Black body emission:** Thermal black body emission off the particle leads to decoherence. This usually becomes relevant for extremely hot particles, and on relatively long timescales [NJP **12**, 033015 (2010)]. In our all-electrical setup, the particles are not constantly illuminated by a laser and therefore do not heat up internally. The spatial decoherence rates due to emission, absorption and scattering of blackbody radiation read [PRA **84**, 052121 (2011)]

$$\Gamma_{\mathrm{em\,(ab)}} = \frac{16\pi^5 c R^3}{189}\left[\frac{k_B T_{i(e)}}{\hbar c}\right]^6 \mathrm{Im}\left[\frac{\epsilon - 1}{\epsilon + 2}\right]\Delta z^2,$$

$$\Gamma_{sc} = \frac{8!\,8\zeta(9) c R^6}{9\pi}\left[\frac{k_B T_e}{\hbar c}\right]^9 \mathrm{Re}\left[\frac{\epsilon - 1}{\epsilon + 2}\right]^2 \Delta z^2,$$

where $T_{i\,(e)}$ is the internal (external) temperature. At room temperature, a spherical 10^6 amu silicon particle with $\epsilon = 11.7 + 0.57i$ yields negligible decoherence with $\Gamma_{em\,(ab)} t \approx 3.9 \cdot 10^{-16}$ and $\Gamma_{sc} t \approx 1.3 \cdot 10^{-19}$. Here we assumed the same absorption coefficient as was used in [PRA **84**, 052121 (2011)].