Macroscopicity of quantum superposition states – Supplementary Material

Stefan Nimmrichter[1] **and Klaus Hornberger**[2]

March 21, 2013

Here we provide a more extensive motivation for the many-body form (5) of the modification, and a justification for our estimates of the macroscopicities reached in the various experiments. We note that a more detailed analysis of the individual experimental setups might yield slightly different values, but would not change the overall picture.

1 Many-particle form of the classicalizing modification

The requirement of invariance under Galileian transformations (i) implies that the generator describing the effect of a minimal modification is of the form

$$\mathcal{L}_N\rho = \int \mathrm{d}^3s\mathrm{d}^3v \left[\mathsf{L}(\boldsymbol{s},\boldsymbol{v})\rho\mathsf{L}^\dagger(\boldsymbol{s},\boldsymbol{v}) - \frac{1}{2}\left\{\mathsf{L}^\dagger(\boldsymbol{s},\boldsymbol{v})\mathsf{L}(\boldsymbol{s},\boldsymbol{v}),\rho\right\}\right]. \tag{S1}$$

The operators $\mathsf{L}(\boldsymbol{s},\boldsymbol{v})$ must satisfy

$$\exp\left[-\frac{i}{\hbar}(\mathbf{P}\cdot\boldsymbol{s}' - M\boldsymbol{v}'\cdot\mathbf{X})\right]\mathsf{L}(\boldsymbol{s},\boldsymbol{v})\exp\left[\frac{i}{\hbar}(\mathbf{P}\cdot\boldsymbol{s}' - M\boldsymbol{v}'\cdot\mathbf{X})\right] = \exp\left[\frac{im_\mathrm{e}}{\hbar}(\boldsymbol{v}\cdot\boldsymbol{s}' - \boldsymbol{v}'\cdot\boldsymbol{s})\right]\mathsf{L}(\boldsymbol{s},\boldsymbol{v}), \tag{S2}$$

with $M = \sum_{n=1}^N m_n$ the total mass, m_e an arbitrary reference mass, and $\mathbf{X}$, $\mathbf{P}$ the center-of-mass position and momentum operators. By switching to center-of-mass and relative coordinates it follows from Eq. (S2) that the $\mathsf{L}(\boldsymbol{s},\boldsymbol{v})$ induce a net shift of the center-of-mass position and momentum by $m_\mathrm{e}\boldsymbol{s}/M$ and $m_\mathrm{e}\boldsymbol{v}$, respectively. However, it remains unspecified how the net shift is to be distributed amongst the N constituents of the system. This freedom is constrained by the additional assumptions (ii)-(iv).

Assumption (iii) means that the N-particle form (S1) must always reduce to the single-particle form for the nth particle, Eq. (1) in the main text, if one traces over the other $N-1$ constituents, $\mathrm{tr}_{N-1}(\mathcal{L}_N\rho) = \mathcal{L}_1\mathrm{tr}_{N-1}(\rho)$. That is to say, we assign to each particle species of mass m_n an individual time parameter τ_n

[1]University of Vienna, Vienna Center for Quantum Science and Technology (VCQ), Boltzmanngasse 5, 1090 Vienna, Austria
[2]University of Duisburg-Essen, Faculty of Physics, Lotharstraße 1, 47048 Duisburg, Germany

and a positive, normalized and isotropic distribution function $g_n(s,q)$. They constitute the free parameters of the single-particle form, as discussed in the main text. At the other end, assumption (iv) recovers the single-particle form (1) for the center-of-mass degree of freedom provided that the N constituents are well localized at fixed equilibrium positions close to the center. We denote the corresponding classicalization parameters by $\tau^{(N)}$ and $g^{(N)}$. Finally, the N-particle operators $\mathsf{L}(\boldsymbol{s},\boldsymbol{v})$ must be symmetrized in the case of indistinguishable particles (ii).

As a first guess one might think that the $\mathsf{L}(\boldsymbol{s},\boldsymbol{v})$ should be proportional to the unitary N-particle Weyl operators,

$$\begin{aligned}\mathsf{L}(\boldsymbol{s},\boldsymbol{v}) &= \sqrt{\frac{m_\mathrm{e}^6}{M^3\tau^{(N)}}g^{(N)}\left(\frac{m_\mathrm{e}}{M}s,m_\mathrm{e}v\right)}\bigotimes_{n=1}^{N}\exp\left[\frac{im_\mathrm{e}}{\hbar M}\left(m_n\boldsymbol{v}\cdot\mathbf{x}_n-\mathbf{p}_n\cdot\boldsymbol{s}\right)\right]\\ &= \sqrt{\frac{m_\mathrm{e}^6}{M^3\tau^{(N)}}g^{(N)}\left(\frac{m_\mathrm{e}}{M}s,m_\mathrm{e}v\right)}\exp\left[\frac{i}{\hbar}\left(m_\mathrm{e}\boldsymbol{v}\cdot\mathbf{X}-\frac{m_\mathrm{e}}{M}\mathbf{P}\cdot\boldsymbol{s}\right)\right].\end{aligned} \tag{S3}$$

This way the phase-space shift would be distributed equally among all participating particles. The assumptions (ii) and (iv) would be fulfilled by construction and, due to the prefactor in (S3), one would reobtain the center-of-mass distribution $g^{(N)}(s,q)$ in (S1). However, the operators (S3) would leave the relative motion of any constituent subsystem entirely unaffected, irrespectively of the overall size and extension of the N-body system. Moreover, assumption (iii) is met only if $\tau^{(N)}=\tau_n$ and $g^{(N)}(s,q)=(m_n/M)^3\,g_n(s,m_nq/M)$ for all n, so that the effective classicalization rate $1/\tau^{(N)}$ would not increase with the system size. Therefore, the operators (S3) cannot induce classical behavior at the macro-scale leaving at the same time microscopic systems unaffected.

Rather than dividing the phase-space shift among many particles, one may as well compose a solution of (S2) from single-particle translations,

$$\mathsf{L}(\boldsymbol{s},\boldsymbol{v})=\sum_{n=1}^{N}(\pm)_n\,\sqrt{\frac{m_\mathrm{e}^6}{m_n^3\tau_n}g_n\left(\frac{m_\mathrm{e}}{m_n}s,m_\mathrm{e}v\right)}\exp\left[\frac{i}{\hbar}\left(m_\mathrm{e}\boldsymbol{v}\cdot\mathbf{x}_n-\frac{m_\mathrm{e}}{m_n}\mathbf{p}_n\cdot\boldsymbol{s}\right)\right]. \tag{S4}$$

The sign $(\pm)_n$ may differ for distinguishable particles, and it may also depend on s and v. These operators fulfill (ii) and (iii) by definition. Moreover, in the case of a compact compound, where $\mathbf{x}_n\approx\mathbf{X}$ and $\mathbf{p}_n\approx m_n\mathbf{P}/M$, we recover condition (iv), with the center-of-mass parameters determined by

$$\sqrt{\frac{m_\mathrm{e}^3}{M^3\tau^{(N)}}g^{(N)}\left(\frac{m_\mathrm{e}}{M}s,q\right)}=\sum_{n=1}^{N}(\pm)_n\,\sqrt{\frac{m_\mathrm{e}^3}{m_n^3\tau_n}g_n\left(\frac{m_\mathrm{e}}{m_n}s,q\right)}. \tag{S5}$$

Each constituent contributes to the collective classicalization of the center-of-mass variables. For instance, in the case of N indistinguishable particles we find that the classicalization rate is amplified by $1/\tau^{(N)}=N^2/\tau$. A formal proof of the operator expression (S4) can be obtained in the picture of second quantization if one takes the operators $\mathsf{L}(\boldsymbol{s},\boldsymbol{v})$ to be a combination of single-particle terms, i.e. a bilinear form in the annihilation

and the creation operators of a given particle species. The N^2-scaling of the classicalization rate then follows immediately.

In general the sign factor $(\pm)_n$ and the single-particle distributions g_n might still differ from species to species. This is where the scale-invariance argument (iv) can be invoked once again: If we allowed for different signs and distribution functions for different point-particle species, we would have to define a fixed set of reference point particles, i.e. single out a distinguished many-body representation of any composite mechanical system. Moreover, we would end up with different descriptions of the modified time evolution of approximate point-like compound particles depending on their composition.

In order to guarantee true scale invariance, by avoiding an ambiguous treatment of approximate point particles, one must therefore relate the extensive nature of the modification in (S5) to the elementary extensive property of mechanical systems: their mass. We notice that the summands in (S5) contribute with *a priori* different mass scales m_n, each of which might be composed of further sub-units of mass, and so on. A unified description for any mass scale is obtained only by introducing a single reference time parameter τ_e and distribution function g_e associated with a fixed reference mass m_e. The composition rule (S5) then holds naturally once we identify

$$(\pm)_n \sqrt{\frac{m_\mathrm{e}^3}{m_n^3\tau_n} g_n\left(\frac{m_\mathrm{e}}{m_n}s, q\right)} = \frac{m_n}{m_\mathrm{e}} \sqrt{\frac{1}{\tau_\mathrm{e}} g_\mathrm{e}\,(s,q)}\,. \tag{S6}$$

The classicalization effect now scales uniformly with mass, irrespectively of the types of particles involved.

2 Diffraction at gratings and double-slits

The macroscopicity observed in the matter-wave diffraction experiments at gratings and double-slits [1–6] can be estimated from the height of the first order diffraction peak in the recorded signals. Given the transmission function $t(x)$ of a one-dimensional N-slit-grating with slit distance d and opening width w, we find the interference signal of a monochromatic point source (velocity v_z) in the paraxial approximation

$$I_{v_z}(x) \propto \int \mathrm{d}x_1 \mathrm{d}x_2\, R\,(x_1 - x_2)\, t(x_1) t^*(x_2)\, \exp\left\{\frac{im}{2\hbar}\left[\frac{x_1^2 - x_2^2}{T} - \frac{2x(x_1 - x_2)}{T_2}\right]\right\}. \tag{S7}$$

The time $T = T_1T_2/(T_1 + T_2)$ is determined by the times-of-flight T_1 from the source to the grating and T_2 from the grating to the screen. In a horizontal alignment they are related to the respective distances $L_{1,2} = v_zT_{1,2}$. The signal (S7) must be averaged with respect to the distributions of the velocities v_z, and over the extensions S and D of the source slit and the detector. The single-particle classicalization as described by equation (1) in the main text blurs the interference signal by the factor

$$R(x) = \exp\left\{\frac{T_1}{\tau}\int_0^1 \mathrm{d}z\left[\widetilde{g}_{1\mathrm{D}}\left(xz, \frac{mx}{T_1}\right) - 1\right] + \frac{T_2}{\tau}\int_0^1 \mathrm{d}z\left[\widetilde{g}_{1\mathrm{D}}\left(xz, \frac{mx}{T_2}\right) - 1\right]\right\}, \tag{S8}$$

involving the reduced Fourier transform $\widetilde{g}_{1\mathrm{D}}(x,p) = \widetilde{g}(x\boldsymbol{e}_x, p\boldsymbol{e}_x)$ of the distribution function g. The unperturbed fringe pattern exhibits diffraction maxima at screen coordinates close to integer multiples of $x = hT_2/md$. The classicalization affects them strongest if the contributing interference paths are completely resolved by the critical length scale $\hbar/\sigma_q$. In this limit a flat background is added to the overall signal, and the longitudinal velocity distribution affects only weakly the reduction of the first diffraction maximum; the latter can thus be used to extract the macroscopicity μ for each experiment according to equation (9) of the main text. The required parameter f is estimated by the ratio of the measured height of the diffraction peak and its unperturbed theoretical value, both normalized to the integrated signal. The following table contains all required parameters of the different experiments, including references to the data used for the comparison.

Ref.	Fig.	L_1/m	L_2/m	N	d/μm	w/d	S/μm	D/μm	$\langle v_z\rangle/\frac{\mathrm{m}}{\mathrm{s}}$	$\Delta v_z/\langle v_z\rangle$	m/amu	f	μ
[1]	9	4.0	5.7	2	107	–	10	30	907	–	1	0.6	4.8
[2]	7	5.0	5.0	2	126	0.17	15	30	216	0.05	1	0.9	6.2
[3]	2b	1.0	1.5	50	0.2	0.5	10	25	1000	0.12	23	0.5	6.8
[4]	3b	0.08	0.11	2	6	0.33	20	20	–	–	20	0.8	9.1
[5]	1	0.45	0.52	100	0.1	0.43	10	25	396	0.1	84	0.8	8.3
[6]	2a	1.14	1.25	100	0.1	0.38	10	8	226	0.6	720	0.6	10.6

The neutron interference at a biprism observed in [1] can be related to the coherent superposition of two virtual sources separated by $d = 107\,\mu$m, 9.7 m away from the detector. The resulting fringe pattern thus resembles a double-slit pattern. The authors of [1] present the measured data and a fitted theory curve in Fig. 9. The data deviates from the predicted height of 200 a.u. of the first diffraction order by roughly 50 a.u. Subtracting a dark count rate of 60 a.u. leads to the estimate $f \sim 1 - 50/140 \approx 0.6$. The experiment [4] is a vertically aligned interferometer, where neon atoms are released from a trap. They fall through a double-slit and into a detector within $t \approx 200$ ms. We obtain f by comparing the measured diffraction peak in Fig. 3b with the theoretical model of [4] in Fig. 3g. For [1–5] we use a Gaussian velocity distribution with the mean $\langle v_z\rangle$ and the FWHM Δv_z, as specified in the table. For [6] we use the distribution provided in the article, and we account for the special detection scheme by replacing the detector slit by a Gaussian laser focus of waist D. The dispersive interaction between the particles and the grating walls is taken into account for [5, 6] by reducing the effective slit opening size.

The macroscopicity of the proposed optical double-slit experiment [7] with silica nanospheres is estimated by considering the Fourier amplitude which corresponds to the expected double-slit fringe oscillation (S7). The classicalization modifies it by the factor $R(d)$, with the largest proposed value for the slit distance $d = 52$ nm. We evaluate the macroscopicity by modeling the particles as homogeneous spheres ($\varrho = 2200\,\mathrm{kg/m^3}$) of 20 nm radius (see equations (6) and (7) of the main text), and by assuming that at least 50% of the fringe amplitude is observed.

Talbot-Lau interference with molecules and clusters [8–12] can be treated in a similar manner. The sinusoidal fringe visibility $\mathcal{V}_{\rm sin}$ of a symmetric setup ($T_1 = T_2 = T$) is reduced to $R\,(hT/md)\,\mathcal{V}_{\rm sin}$. Judging from the error bars at high visibilities, we assume that the measurements are compatible with at least 90% of the prediction for C_{70} molecules (Fig. 3 in [8]), 90% for $C_{60}F_{48}$ (Fig. 5 in [10]), and 80% for PFNS8 (Fig. 4b in [11]). The fringe pattern observed in [9] with $C_{60}F_{48}$ (Fig. 4) corresponds to 75% of the predicted visibility.

3 Ramsey-Bordé interference with I_2 molecules

In the experiment [13] a beam of I_2 molecules ($m = 254\,$amu) passes two pairs of counter-propagating running-wave laser beams, as described in detail in [14]. Two paths through the setup contribute to the recorded Ramsey fringe pattern, as shown in Fig. 2 of Ref. [13]. We must include, however, a significant contribution from two further paths to the signal [14]. Their interference is washed out over the transverse velocity distribution of the molecule beam. We assume that they add an offset to the most pronounced central fringe of the (*)-curve in Fig. 2 of [13]. If all four paths contribute by roughly the same weight we must halve the offset of the central fringe in the diagram, which yields a two-path fringe visibility of $f \approx 400\,\text{a.u.}/(2400 - 1200)\,\text{a.u.} = 0.33$. The passage time is determined by the $(35 + 2)\,$mm length of the interferometer and the mean molecular velocity of 350 m/s. This yields $\mu = 7.3$, according to equation (9) of the main text.

4 Mach-Zehnder-type interference

The two atom interferometers featuring the greatest macroscopicity in Fig. 2 [15,16], as well as the proposed satellite atom interferometer [17] listed in Tab. 1, are optical Mach-Zehnder-type geometries, which could in principle yield close to 100% fringe contrast. We use the recorded fringe visibilities $f = 0.62$ (Fig. 19 in [15]) and $f = 0.33$ (Fig. 3 in [16]), and a hypothetical value of $f = 0.5$ for the proposal [17]. In all three cases the interfering particles are ^{133}Cs atoms, and the interrogation time is given by twice the pulse separation time T. The respective values are $T = 160\,$ms, 400 ms, and 2000 s.

In the Na_2 molecule interferometer [18] the Mach-Zehnder geometry is realized with three material diffraction gratings. The total length of the interferometer including beam collimation is about 2.1 m [19,20], and the molecules pass it at a mean velocity of 820 m/s. This yields an interrogation time of 2.6 ms. The maximally possible contrast is limited by two factors: First, the different weights of the interference paths, which correspond to the zeroth and the first diffraction order at the first grating; they are given by $P_1/P_0 = \text{sinc}\,(0.3\pi)\,/1 = 0.74$. Second, the modulation of the interference pattern by the third grating mask; it contributes a factor of $\text{sinc}\,(0.3\pi) = 0.86$ to the fringe amplitude, assuming a sinusoidal fringe pattern and a

grating opening fraction of 30%. The detected contrast is thus limited to below 85%. We extract a measured fringe contrast of about 30% from the inset of Fig. 4 in [18], i.e. $f = 0.35$, which leads to $\mu = 7.2$.

5 Oscillating microresonators

The authors of [21] propose to create a quantum superposition state of an oscillating micromirror by entangling it with a single cavity photon in one arm of a Michelson interferometer. If coherence is maintained in the mirror motion during one period of oscillation photon interference fringes should be observed at 100% contrast.

The classicalization master equation (equation (1) in the main text) for the harmonic mirror motion can be integrated explicitly. We find that it reduces the fringe visibility after one oscillation period $2\pi/\omega_m$ by the factor

$$R = \exp\left\{\int_0^{2\pi} \frac{\mathrm{d}\xi}{\omega_m \tau}\left[\widetilde{g}_{1\mathrm{D}}\left(2\kappa x_0 \sin^2\frac{\xi}{2}, \frac{2\hbar\kappa}{x_0}\sin\xi\right) - 1\right]\right\} \tag{S9}$$

with $\widetilde{g}_{1\mathrm{D}}$ the reduced Fourier transform of the distribution g. The latter and the time parameter τ are given by equations (6) and (7) in the main text. The micromirror is modeled as a homogeneous cube of mass density $\varrho = 2300\,\mathrm{kg/m^3}$, $b = 10\,\mu$m edge length and a mass of $M = \varrho b^3 = 2.3\,$ng, which yields

$$\frac{1}{\tau} = \frac{1}{\tau_e}\left(\frac{M}{m_e}\right)^2 \gamma^3, \tag{S10}$$

$$g_{1\mathrm{D}}(s,q) = \gamma^{-1}\frac{M}{2\pi m_e \sigma_s \sigma_q}\exp\left(-\frac{M^2 s^2}{2m_e^2\sigma_s^2} - \frac{q^2}{2\sigma_q^2}\right)\mathrm{sinc}^2\left(\frac{qb}{2\hbar}\right), \tag{S11}$$

with the σ_q-dependent geometry factor

$$\gamma = 2\left(\frac{\sigma_q b}{\hbar}\right)^{-2}\left[\exp\left(-\frac{\sigma_q^2 b^2}{2\hbar^2}\right) + \sqrt{\frac{\pi}{2}}\frac{\sigma_q b}{\hbar}\,\mathrm{erf}\left(\frac{\sigma_q b}{\sqrt{2}\hbar}\right) - 1\right]. \tag{S12}$$

The authors presume a frequency $\omega_m/2\pi = 500\,$Hz, a ground state oscillation amplitude of $x_0 = 170\,$fm and a photon-mirror coupling strength of $\kappa = 1.63$. We find $\mu = 19.0$ for a measured 50% fidelity.

For the hypothetical superposition experiment with an oscillating Al micromembrane, as listed in Table I of the main text, we use the parameters given in [22]. The membrane mass $M = 48\,$pg and the mechanical frequency $\omega_m/2\pi = 10.56\,$MHz yield a tiny ground state amplitude of $x_0 = \sqrt{2\hbar/M\omega_m} = 8\,$fm. To give a good upper estimate of the macroscopicity of such an experiment we thus approximate the flexural mode of the membrane by a axial center-of-mass vibration of a homogeneous disc of thickness $b = 100\,$nm and radius $R = 7.5\,\mu$m. We obtain the effective distribution $g_{1\mathrm{D}}(s,q)$ of (S11) and a time parameter

$$\frac{1}{\tau} = \frac{2\gamma}{\tau_e}\left(\frac{M}{m_e}\right)^2\left(\frac{\sigma_q R}{\hbar}\right)^{-2}\exp\left(-\frac{\sigma_q^2 R^2}{\hbar^2}\right)\left[\exp\left(\frac{\sigma_q^2 R^2}{\hbar^2}\right) - I_0\left(\frac{\sigma_q^2 R^2}{\hbar^2}\right) - I_1\left(\frac{\sigma_q^2 R^2}{\hbar^2}\right)\right], \tag{S13}$$

with $I_{0,1}$ the modified Bessel functions.

We assume that the membrane is prepared in the superposition state $|\psi\rangle = (|0\rangle + |1\rangle) / \sqrt{2}$ of the zero- and the one-phonon eigenstate, and that the associated nondiagonal matrix element $\langle 1|\rho_t|0\rangle$ does not decay by more than 50% after a time $t = 2\pi n/\omega_m$ which corresponds to $n = 1000$ oscillation cycles. Due to the large mass M and the small amplitude x_0 we may neglect the position spread σ_s in (S11), and we may Taylor-expand its Fourier transform $\widetilde{g}_{1\mathrm{D}}(x, p)$ to lowest order in x. With this we arrive at the explicit condition

$$\frac{\langle 1|\rho_t|0\rangle}{\langle 1|\psi\rangle\langle\psi|0\rangle} = \left(\frac{2\pi n}{\omega_m \tau}\frac{x_0^2}{\gamma b^2}\left[1 - \exp\left(-\frac{\sigma_q^2 b^2}{2\hbar^2}\right)\right] + 1\right)^{-2} \geq 50\%, \tag{S14}$$

which leads to a macroscopicity of $\mu = 11.5$.

6 BEC interference

The interference of two sodium BECs observed in [23] is modeled using a second quantization phase-space picture in [24]. Following the same line we define a second quantization form of the characteristic function,

$$\hat{\chi}(x, p) = \int \mathrm{d}x_0\, e^{ipx_0/\hbar} \hat{\psi}^\dagger\left(x_0 + \frac{x}{2}\right)\hat{\psi}\left(x_0 - \frac{x}{2}\right). \tag{S15}$$

De Broglie interference of trapped BECs is observed as a fringe pattern in the time-evolved single-particle density $\hat{n}(x) = \hat{\psi}^\dagger(x)\hat{\psi}(x)$ for each individual run of the experiment. The fringe visibility is given by the corresponding Fourier component of $\hat{n}(x)$,

$$\hat{\chi}\left(0, \frac{h}{\lambda}\right) = \int \mathrm{d}x_0\, e^{2\pi i x_0/\lambda} \hat{n}(x_0), \tag{S16}$$

where λ denotes the fringe spacing. The pattern observed in each run of the experiment [23] can be assessed in the case of non-interacting bosons by replacing the annihilation operator $\hat{\psi}(x)$ with the collective wave function $\psi(x)$ of the two trapped condensates. A free evolution of (S15) by the time t then yields the visibility $\chi(0, h/\lambda)$ of the resulting interference pattern. The fringe spacing $\lambda =$ should be modified to account for interactions in the BEC [24].

The second quantization form of the reduced N-particle operators (4) reads as

$$\mathsf{W}(s, q) = \frac{m}{m_e}\int \mathrm{d}x\, e^{-iqx/\hbar} \hat{\psi}^\dagger(x)\,\hat{\psi}\left(x + \frac{m_e}{m}s\right). \tag{S17}$$

A straightforward calculation reveals that (S15) then classicalizes at the rate

$$\mathcal{L}\hat{\chi}(x, p) = -\left(\frac{m}{m_e}\right)^2 \frac{1}{\tau_e}\left[1 - \frac{m}{m_e}\int \mathrm{d}s\,\mathrm{d}q\; g_e\left(\frac{m}{m_e}s, q\right) e^{i(qx - ps)/\hbar}\right]\hat{\chi}(x, p) \tag{S18}$$

of a single atom. We therefore estimate the macroscopicity from equation (9) of the main text. The authors of [23] observed about $f = 75\%$ interference contrast in a sodium BEC after a time-of-flight of $t = 40\,\mathrm{ms}$;

this yields $\mu = 8.4$. In the experiment [25] the phase sensitivity of the interferometer was increased, but at an interference contrast of only 15% after 200 ms, which leads to $\mu = 8.3$ (not discussed in the main text).

Modern-day experiments with multi-component BECs make use of nonlinear interactions and number squeezing to increase the coherence time and the phase sensitivity employing internal atomic states [26–28]. The use of such techniques in interference experiments with spatially split BECs would only yield a macroscopicity μ that is comparable to single-atom interferometers. This is due to the fact that the single-particle nature of the classicalizing effect (S18) holds irrespectively of whether nonlinear interactions modify the coherent time evolution of the condensate wavefunction. Larger values of μ could be achieved by increasing the fringe visibility and the time-of-flight in both single-atom and BEC experiments, possibly carried out in a microgravity environment.

7 SQUID interference

For the case of SQUID experiments we obtain the exclusion curve of the classicalization parameters in Fig. 1 and the μ-values in Fig. 2 and Tab. 1 by estimating the decay rate of a superposition state of macroscopically different supercurrents, i.e. different phases across the junctions in a Josephson loop. This was studied theoretically for spontaneous localization models in [29], whose observable consequences are a special case of the classicalizing modification discussed here [30].

A state of finite current density $|\boldsymbol{j}\rangle$ in a solid with electron density n_e is described by a Fermi sphere, displaced by the momentum $\hbar\boldsymbol{k}_{\boldsymbol{j}}$, $\boldsymbol{j} = n_e e\hbar\boldsymbol{k}_{\boldsymbol{j}}/m_e$. The undisplaced state $|0\rangle$ is taken to be the BCS ground state of the superconductor [31, 32]. It is characterized by the probability amplitudes $v_{\boldsymbol{k}}$ ($u_{\boldsymbol{k}} = \sqrt{1 - v_{\boldsymbol{k}}^2}$) of a Cooper pair ($\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow$) being occupied (unoccupied),

$$v_{\boldsymbol{k}} = \frac{1}{2}\left(1 - \frac{k^2 - k_F^2}{\sqrt{\left(k^2 - k_F^2\right)^2 + \left(2m_e\Delta_{\boldsymbol{k}}/\hbar^2\right)^2}}\right). \tag{S19}$$

Here, $\hbar k_F = m_e v_F$ denotes the Fermi momentum and $\Delta_{\boldsymbol{k}}$ the pairing energy. The latter is approximated in the usual way by the zero-temperature energy gap $\Delta = 1.76 k_B T_c$ for electrons close to the Fermi level, $|k^2 - k_F^2| \leq 2m_e\omega_D/\hbar$, and zero otherwise. The term ω_D denotes the Debye cutoff frequency of the material. We use the literature values $k_F = 1.18\,\text{Å}^{-1}$, $\Delta = 1.44\,\text{meV}$, $\hbar\omega_D = 23.7\,\text{meV}$ for Nb, and $k_F = 1.74\,\text{Å}^{-1}$, $\Delta = 0.17\,\text{meV}$, $\hbar\omega_D = 36.9\,\text{meV}$ for Al, respectively [33, 34].

The second quantization form of the classicalization operators (equation (4) in the main text) for electrons reads as

$$\mathsf{W}_e(\boldsymbol{s}, \hbar\boldsymbol{q}) = \sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{s}}\mathsf{a}_\sigma^\dagger(\boldsymbol{k})\,\mathsf{a}_\sigma(\boldsymbol{k}+\boldsymbol{q}). \tag{S20}$$

The sum covers all discrete electron momentum states in a given volume V, each state occupying the elementary cell $(2\pi\hbar)^3/V$ in momentum space. The classicalization kick distribution $g_e(s,q)$ must be discretized accordingly. We find that a superposition state ρ of two distinct supercurrents $\boldsymbol{j}_1$ and $\boldsymbol{j}_2$ decays at a rate

$$\Gamma \approx -\frac{\partial_t \langle \boldsymbol{j}_1|\rho(t)|\boldsymbol{j}_2\rangle}{\langle \boldsymbol{j}_1|\rho(t)|\boldsymbol{j}_2\rangle}\bigg|_{t=0} = \Gamma_{\text{diff}} + \Gamma_{\text{deph}} \tag{S21}$$

due to classicalization. This assumes that $\langle \boldsymbol{j}_1|\boldsymbol{j}_2\rangle = 0$, and that the net number δN of electrons occupying different states in each superposition branch [34] is large. The decay rate splits into two contributions. The first one is related to momentum diffusion, which requires that at least one elementary unit of momentum $2\pi\hbar/V^{1/3}$ is transferred. In the continuum limit $\sum_{\boldsymbol{k}} \to V/(2\pi)^3 \int \mathrm{d}^3k$ we find

$$\Gamma_{\text{diff}} = \frac{2V\hbar^3}{(2\pi)^3\tau_e} \iiint_{q>\pi/V^{1/3}} \mathrm{d}^3s\, \mathrm{d}^3q\, \mathrm{d}^3k\; g_e(s,\hbar q)\; u_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{q}} \left(u_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{q}} + v_{\boldsymbol{k}} u_{\boldsymbol{k}+\boldsymbol{q}} e^{i(2\boldsymbol{k}+\boldsymbol{q})\cdot\boldsymbol{s}}\right). \tag{S22}$$

The expression is ultimately bounded by $\Gamma_{\text{diff}} \leq N/\tau_e$ in the limit of arbitrarily strong momentum kicks, when all $N = n_e V$ conducting electrons can be transferred from one branch of the superposition to the other. It does not depend on the actual value of the supercurrents.

The second contribution represents the dephasing that comes from the classicalization-induced position kicks, when no momentum redistribution of the electrons takes place,

$$\Gamma_{\text{deph}} = \frac{4V^2\hbar^3}{(2\pi)^6\tau_e} \iint_{q\leq\pi/V^{1/3}} \mathrm{d}^3s\, \mathrm{d}^3q\; g_e(s,\hbar q)\left(1 - e^{i\delta\boldsymbol{k}\cdot\boldsymbol{s}}\right)\left|\int \mathrm{d}^3k\; v_{\boldsymbol{k}}^2\, e^{i\boldsymbol{k}\cdot\boldsymbol{s}}\right|^2. \tag{S23}$$

Here, $\hbar\delta\boldsymbol{k} = m_e(\boldsymbol{j}_1 - \boldsymbol{j}_2)/n_e e$ denotes the difference in momentum displacement of the two current branches. It is orders of magnitude smaller than the Fermi momentum, and $|\delta\boldsymbol{k}\cdot\boldsymbol{s}| \ll 1$ holds for any reasonable kick distribution g_e. Hence the dephasing contribution scales quadratically with the net difference in occupation of the two displaced Fermi spheres, $\delta N = 4N|\delta\boldsymbol{k}|/3k_F$. While this may potentially be significant for large SQUID geometries, the diffusion contribution dominates in all existing real-size experiments.

Experimentally measured coherence times T_2 of such current superpositions provide an upper bound for the decay rate Γ. We estimate T_2 by the smallest observed frequency splitting in the experiments [35] ($T_2 \approx 1\,\text{ns}$) and [36] ($T_2 \approx 10\,\text{ns}$); the authors of [37] estimate $T_2 \approx 15\,\text{ns}$. Classicalization parameters which lead to $\Gamma > 1/T_2$ are then excluded by each experiment. This yields the SQUID curve in Fig. 1, as well as the μ-values plotted in Fig. 2; the latter are computed with the boundary condition $\sigma_s \leq 1\ \text{Å} \leq \hbar/\sigma_q$, as discussed in the main text. The superconducting loop is spanned by $L = 560\,\mu\text{m}$ of Nb in [35], $20\,\mu\text{m}$ of Al in [37], and $180\,\mu\text{m}$ of Al in [36]. We assume the respective material cross-sections as $5\,\mu\text{m}^2$, $36000\,\text{nm}^2$, and $1\,\mu\text{m}^2$. The experiment [37] yields a smaller macroscopicity, $\mu = 3.3$, than [35] ($\mu = 5.2$) due to its smaller ring geometry. Only the greater value is included in Fig. 2. The large hypothetical SQUID in Tab. 1 of the main text is a 20 mm loop of $100\,\mu\text{m}^2$ cross-section with a coherence time of 1 ms.

The actual values of the supercurrents do not influence the μ-values, since the dephasing contribution is negligible in all cases. We use a current difference of $I_1 - I_2 = 3\,\mu$A for Fig. 1, as given in [34].

8 Schrödinger's gedankenexperiment

In our version of the famous gedankenexperiment, as listed in Tab. 1 of the main text, we consider the hypothetical superposition state of an ideal cat sitting at two places $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ that are 10 cm apart. The center-of-mass coherence of the cat then decays like

$$\frac{\partial_t \langle \boldsymbol{x}_1|\rho|\boldsymbol{x}_2\rangle}{\langle \boldsymbol{x}_1|\rho|\boldsymbol{x}_2\rangle} = \frac{1}{\tau}\int \mathrm{d}^3 s\, \mathrm{d}^3 q\; g\,(s,\boldsymbol{q})\left(e^{i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)/\hbar} - 1\right) \tag{S24}$$

due to classicalization. We have neglected the weak position diffusion in the classicalization master equation (1) here. The mean coherence time of this state shall be 1 s. In order to evaluate the above decay rate using the τ and the g of a compound, as defined by (6) and (7) in the main text, we model the cat as a homogeneous sphere of water with a mass of 4 kg.

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